Deriving the long term value of an appreciating asset

Base math formula for sum of a geometric progression: $\sum_{t=0}^{\infty} a_0 r^t = \frac{a_0}{1-r}$

Infinite horizon

$$\sum_{t=0}^{\infty} v \frac{(1+g)^t}{(1+d)^t} = \sum_{t=0}^{\infty} v \left(\frac{1+g}{1+d}\right)^t \qquad \text{[so use } r = \frac{1+g}{1+d} \text{ in base formula]}$$

$$= v \frac{1}{1-\frac{1+g}{1+d}} = v \frac{1+d}{1+d-1-g} = v \frac{1+d}{d-g}$$

Finite horizon

$$\sum_{t=0}^{T} v \frac{(1+g)^t}{(1+d)^t} = v \left[\sum_{t=0}^{\infty} \left(\frac{1+g}{1+d}\right)^t - \sum_{t=T+1}^{\infty} \left(\frac{1+g}{1+d}\right)^t \right]$$

$$= v \left[\left(\frac{1+d}{d-g}\right) - \left(\frac{1+g}{1+d}\right)^{T+1} \left(\frac{1+d}{d-g}\right) \right]$$

$$= v \left(\frac{1+d}{d-g}\right) \left[1 - \left(\frac{1+g}{1+d}\right)^{T+1} \right]$$