Chapter 3 "Drawing from a Reservior" model

dynamic parameters

```
T = 40;
d = .06;
g = .01;
```

benefits (build demand using parameters from Chap 2 and integrate under it)

```
q0 = 35 000;
p0 = 3;
elast = -1 / 2;
c = q0 / p0^elast;
wlog = (1 + g) ^ (t - 1) * c * p^elast;
mb = p /. Flatten[Simplify[Solve[w == %, p]]]
```

$$\frac{3.60259\times 10^{9}\ e^{0.0199007\,t}}{w^{2}}$$

```
Assuming[{W > 1000}, tb = ∫_1000^W mb dw]
```

$$\frac{3.60259\times 10^{6}\ e^{0.0199007\,t}\ (-1000. + W)}{W}$$

costs

```
tc = 50 000 + 1.95 * W + 0.000015 * W^2;
```

hydrologics

```
initialstored = 160 000;
annualflow = 27 000;
```

develop work matrix

```
wvector = Prepend[Array[w, T - 1], w0];
work = Table[1., {t, T}, {j, 6}];
work[[1, 1]] = 0;
Do[work[[t + 1, 1]] = t, {t, T - 1}]
work[[1, 2]] = 35 000;
Do[work[[t, 2]] = W /. Flatten[Solve[D[(tb - tc), W] == 0, W, Reals]], {t, 2, T}]
Do[work[[t, 3]] = wvector[[t]], {t, T}]
Do[work[[t, 4]] = (tb - tc) /. W → work[[t, 3]], {t, T}]
Do[work[[t, 5]] = work[[t, 4]] / (1 + d) ^ (t - 1), {t, T}]
work[[1, 6]] = initialstored + annualflow;
Do[work[[t, 6]] = work[[t - 1, 6]] + annualflow - work[[t - 1, 3]], {t, 2, T}]
TableForm[
 Prepend[work, {"Period", "MNB=0@", "w", "NBs", "PV(NBs)", "Reserv@Start"}]]
Dual = D[work[[1, 4]], w0];
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

General::stop : Further output of Solve::ratnz will be suppressed during this calculation. ≫

Period	MNB=0@	w	NBs
0	35 000	w_0	$-50\,000 + \frac{3.675\times10^6\,(-1000.+w_0)}{w_0} - 1.95\,w_0 - 0.000015\,w_0^2$
1	35 297.5	w[1]	$-50\,000 + \frac{3.74887\times10^6\,(-1000.+w[1])}{w[1]} - 1.95\,w[1] - 0.000015\,w[1]^2$
2	35 597.3	w[2]	$-50\,000 + \frac{3.82422\times10^6\,(-1000.+w[2])}{w[2]} - 1.95\,w[2] - 0.000015\,w[2]^2$
3	35 899.4	w[3]	$-50\,000 + \frac{3.90109\times10^6\,(-1000.+w[3])}{w[3]} - 1.95\,w[3] - 0.000015\,w[3]^2$
4	36 203.9	w[4]	$-50\,000 + \frac{3.9795\times10^6\,(-1000.+w[4])}{w[4]} - 1.95\,w[4] - 0.000015\,w[4]^2$
5	36 510.6	w[5]	$-50\,000 + \frac{4.05949\times10^6\,(-1000.+w[5])}{w[5]} - 1.95\,w[5] - 0.000015\,w[5]^2$
6	36 819.7	w[6]	$-50\,000 + \frac{4.14108\times10^6\,(-1000.+w[6])}{w[6]} - 1.95\,w[6] - 0.000015\,w[6]^2$
7	37 131.2	w[7]	$-50\,000 + \frac{4.22432\times10^6\,(-1000.+w[7])}{w[7]} - 1.95\,w[7] - 0.000015\,w[7]^2$
8	37 445.	w[8]	$-50\,000 + \frac{4.30923\times10^6\,(-1000.+w[8])}{w[8]} - 1.95\,w[8] - 0.000015\,w[8]^2$
9	37 761.2	w[9]	$-50\,000 + \frac{4.39584\times10^6\,(-1000.+w[9])}{w[9]} - 1.95\,w[9] - 0.000015\,w[9]^2$
10	38 079.8	w[10]	$-50\,000 + \frac{4.4842\times10^6\,(-1000.+w[10])}{w[10]} - 1.95\,w[10] - 0.000015\,w[10]^2$
11	38 400.9	w[11]	$-50\,000 + \frac{4.57433\times10^6\,(-1000.+w[11])}{w[11]} - 1.95\,w[11] - 0.000015\,w[11]^2$
12	38 724.4	w[12]	$-50\,000 + \frac{4.66627\times10^6\,(-1000.+w[12])}{w[12]} - 1.95\,w[12] - 0.000015\,w[12]^2$
13	39 050.3	w[13]	$-50\,000 + \frac{4.76007\times10^6\,(-1000.+w[13])}{w[13]} - 1.95\,w[13] - 0.000015\,w[13]^2$
14	39 378.7	w[14]	$-50\,000 + \frac{4.85574\times10^6\,(-1000.+w[14])}{w[14]} - 1.95\,w[14] - 0.000015\,w[14]^2$

15	39 709.6	w[15]	$-50000+\frac{4.95334\times10^{6}\,(-1000.+w[15])}{w[15]}-1.95\,w[15]-0.000015\,w[15]^{2}$
16	40 043.	w[16]	$-50000+\frac{5.05291\times10^{6}\,(-1000.+w[16])}{w[16]}-1.95\,w[16]-0.000015\,w[16]^{2}$
17	40 378.9	w[17]	$-50000+\frac{5.15447\times10^{6}\,(-1000.+w[17])}{w[17]}-1.95\,w[17]-0.000015\,w[17]^{2}$
18	40 717.4	w[18]	$-50000+\frac{5.25808\times10^{6}\,(-1000.+w[18])}{w[18]}-1.95\,w[18]-0.000015\,w[18]^{2}$
19	41 058.4	w[19]	$-50000+\frac{5.36376\times10^{6}\,(-1000.+w[19])}{w[19]}-1.95\,w[19]-0.000015\,w[19]^{2}$
20	41 401.9	w[20]	$-50000+\frac{5.47157\times10^{6}\,(-1000.+w[20])}{w[20]}-1.95\,w[20]-0.000015\,w[20]^{2}$
21	41 748.1	w[21]	$-50000+\frac{5.58155\times10^{6}\,(-1000.+w[21])}{w[21]}-1.95\,w[21]-0.000015\,w[21]^{2}$
22	42 096.9	w[22]	$-50000+\frac{5.69374\times10^{6}\,(-1000.+w[22])}{w[22]}-1.95\,w[22]-0.000015\,w[22]^{2}$
23	42 448.3	w[23]	$-50000+\frac{5.80819\times10^{6}\,(-1000.+w[23])}{w[23]}-1.95\,w[23]-0.000015\,w[23]^{2}$
24	42 802.3	w[24]	$-50000+\frac{5.92493\times10^{6}\,(-1000.+w[24])}{w[24]}-1.95\,w[24]-0.000015\,w[24]^{2}$
25	43 159.	w[25]	$-50000+\frac{6.04402\times10^{6}\,(-1000.+w[25])}{w[25]}-1.95\,w[25]-0.000015\,w[25]^{2}$
26	43 518.3	w[26]	$-50000+\frac{6.16551\times10^{6}\,(-1000.+w[26])}{w[26]}-1.95\,w[26]-0.000015\,w[26]^{2}$
27	43 880.4	w[27]	$-50000+\frac{6.28943\times10^{6}\,(-1000.+w[27])}{w[27]}-1.95\,w[27]-0.000015\,w[27]^{2}$
28	44 245.1	w[28]	$-50000+\frac{6.41585\times10^{6}\,(-1000.+w[28])}{w[28]}-1.95\,w[28]-0.000015\,w[28]^{2}$
29	44 612.6	w[29]	$-50000+\frac{6.54481\times10^{6}\,(-1000.+w[29])}{w[29]}-1.95\,w[29]-0.000015\,w[29]^{2}$
30	44 982.8	w[30]	$-50000+\frac{6.67636\times10^{6}\,(-1000.+w[30])}{w[30]}-1.95\,w[30]-0.000015\,w[30]^{2}$
31	45 355.8	w[31]	$-50000+\frac{6.81056\times10^{6}\,(-1000.+w[31])}{w[31]}-1.95\,w[31]-0.000015\,w[31]^{2}$
32	45 731.6	w[32]	$-50000+\frac{6.94745\times10^{6}\,(-1000.+w[32])}{w[32]}-1.95\,w[32]-0.000015\,w[32]^{2}$
33	46 110.1	w[33]	$-50000+\frac{7.08709\times10^{6}\,(-1000.+w[33])}{w[33]}-1.95\,w[33]-0.000015\,w[33]^{2}$
34	46 491.5	w[34]	$-50000+\frac{7.22954\times10^{6}\,(-1000.+w[34])}{w[34]}-1.95\,w[34]-0.000015\,w[34]^{2}$
35	46 875.7	w[35]	$-50000+\frac{7.37486\times10^{6}\,(-1000.+w[35])}{w[35]}-1.95\,w[35]-0.000015\,w[35]^{2}$
36	47 262.8	w[36]	$-50000+\frac{7.52309\times10^{6}\,(-1000.+w[36])}{w[36]}-1.95\,w[36]-0.000015\,w[36]^{2}$
37	47 652.8	w[37]	$-50000+\frac{7.6743\times10^{6}\,(-1000.+w[37])}{w[37]}-1.95\,w[37]-0.000015\,w[37]^{2}$
38	48 045.6	w[38]	$-50000+\frac{7.82856\times10^{6}\,(-1000.+w[38])}{w[38]}-1.95\,w[38]-0.000015\,w[38]^{2}$
39	48 441.3	w[39]	$-50000+\frac{7.98591\times10^{6}\,(-1000.+w[39])}{w[39]}-1.95\,w[39]-0.000015\,w[39]^{2}$

develop NPV and single constraint

```
npv = Apply[Plus, Part[Transpose[work], 5]];
```

The Maximize command employed later requires a tedious list of positivity constraints for water use in every period. Below, string commands are employed to build this list of constraints to fit whatever T is and avoid typing them all out when the Maximize command is used.

```
summeduse = Apply[Plus, wvector];
excesswater = initialstored + T * annualflow - summeduse;
positivity = ToString[""];
For[i = 1, i < T, i++,
 positivity = StringJoin[positivity, "&&w[", ToString[i], "]≥0"]]
allconstraints = StringJoin["excesswater==0&&w0>0", ToExpression["positivity"]];
```

Max NPV

```
answer = Maximize[{npv, ToExpression[allconstraints]}, wvector]
```

```
{7.21323 × 10^7, {w0 → 31573.2, w[1] → 31673.3, w[2] → 31765.8, w[3] → 31850.,
  w[4] → 31925.9, w[5] → 31993., w[6] → 32050.7, w[7] → 32099.2, w[8] → 32137.8,
  w[9] → 32166.2, w[10] → 32184.2, w[11] → 32191.4, w[12] → 32187.6, w[13] → 32172.4,
  w[14] → 32145.9, w[15] → 32107.5, w[16] → 32057.1, w[17] → 31994.7,
  w[18] → 31919.9, w[19] → 31832.9, w[20] → 31733.3, w[21] → 31621.3,
  w[22] → 31497., w[23] → 31360.2, w[24] → 31211., w[25] → 31049.5, w[26] → 30876.1,
  w[27] → 30690.8, w[28] → 30493.8, w[29] → 30285.5, w[30] → 30066.1, w[31] → 29836.,
  w[32] → 29595.6, w[33] → 29345.2, w[34] → 29085.2, w[35] → 28816.1,
  w[36] → 28538.4, w[37] → 28252.5, w[38] → 27959.1, w[39] → 27658.6}}
```

incorporate optimal use schedule into "work" matrix and prepare for printing/plotting

```
ans = Flatten[answer];
w0 = w0 /. ans[[2]];
Do[w[i] = w[i] /. ans[[i + 2]], {i, 1, T - 1}]
```

```
TableForm[
 Prepend[work, {"Period", "MNB=0@", "w", "NBs", "PV(NBs)", "Reserv@Start"}]]
```

Period	MNB=0@	w	NBs	PV(NBs)	Reserv@Start
0	35 000	31 573.2	3.43208×10^6	3.43208×10^6	187 000
1	35 297.5	31 673.3	3.5037×10^6	3.30537×10^6	182 427.
2	35 597.3	31 765.8	3.57675×10^6	3.1833×10^6	177 753.
3	35 899.4	31 850.	3.65128×10^6	3.06568×10^6	172 988.
4	36 203.9	31 925.9	3.72731×10^6	2.95238×10^6	168 138.
5	36 510.6	31 993.	3.80486×10^6	2.84321×10^6	163 212.
6	36 819.7	32 050.7	3.88397×10^6	2.73805×10^6	158 219.
7	37 131.2	32 099.2	3.96467×10^6	2.63673×10^6	153 168.
8	37 445.	32 137.8	4.04698×10^6	2.53912×10^6	148 069.
9	37 761.2	32 166.2	4.13094×10^6	2.4451×10^6	142 931.
10	38 079.8	32 184.2	4.21657×10^6	2.35451×10^6	137 765.
11	38 400.9	32 191.4	4.30392×10^6	2.26725×10^6	132 581.
12	38 724.4	32 187.6	4.393×10^6	2.18319×10^6	127 389.
13	39 050.3	32 172.4	4.48385×10^6	2.1022×10^6	122 202.
14	39 378.7	32 145.9	4.57651×10^6	2.02419×10^6	117 029.
15	39 709.6	32 107.5	4.671×10^6	1.94904×10^6	111 883.
16	40 043.	32 057.1	4.76736×10^6	1.87665×10^6	106 776.
17	40 378.9	31 994.7	4.86562×10^6	1.80692×10^6	101 719.
18	40 717.4	31 919.9	4.96582×10^6	1.73974×10^6	96 724.1
19	41 058.4	31 832.9	5.06799×10^6	1.67504×10^6	91 804.2
20	41 401.9	31 733.3	5.17217×10^6	1.61271×10^6	86 971.3
21	41 748.1	31 621.3	5.27838×10^6	1.55266×10^6	82 237.9
22	42 096.9	31 497.	5.38667×10^6	1.49483×10^6	77 616.7
23	42 448.3	31 360.2	5.49707×10^6	1.43912×10^6	73 119.7
24	42 802.3	31 211.	5.60962×10^6	1.38546×10^6	68 759.5
25	43 159.	31 049.5	5.72436×10^6	1.33377×10^6	64 548.5
26	43 518.3	30 876.1	5.84131×10^6	1.28398×10^6	60 499.
27	43 880.4	30 690.8	5.96053×10^6	1.23602×10^6	56 622.9
28	44 245.1	30 493.8	6.08204×10^6	1.18983×10^6	52 932.1
29	44 612.6	30 285.5	6.20589×10^6	1.14534×10^6	49 438.3
30	44 982.8	30 066.1	6.33212×10^6	1.10249×10^6	46 152.8
31	45 355.8	29 836.	6.46076×10^6	1.06121×10^6	43 086.7
32	45 731.6	29 595.6	6.59185×10^6	1.02146×10^6	40 250.7
33	46 110.1	29 345.2	6.72544×10^6	983 167.	37 655.1
34	46 491.5	29 085.2	6.86157×10^6	946 290.	35 310.
35	46 875.7	28 816.1	7.00028×10^6	910 773.	33 224.8
36	47 262.8	28 538.4	7.14161×10^6	876 567.	31 408.7
37	47 652.8	28 252.5	7.28561×10^6	843 623.	29 870.3
38	48 045.6	27 959.1	7.43231×10^6	811 897.	28 617.8
39	48 441.3	27 658.6	7.58177×10^6	781 343.	27 658.6

```
Dual
```

```
0.789341
```

check to see how much water is used

```
summeduse
```

1.24×10^{6}

extract plottable vectors from matrix

```
satiationPts = Transpose[Take[Transpose[work], 2]]
```

```
{{0, 35 000}, {1, 35 297.5}, {2, 35 597.3}, {3, 35 899.4}, {4, 36 203.9},
 {5, 36 510.6}, {6, 36 819.7}, {7, 37 131.2}, {8, 37 445.}, {9, 37 761.2},
 {10, 38 079.8}, {11, 38 400.9}, {12, 38 724.4}, {13, 39 050.3}, {14, 39 378.7},
 {15, 39 709.6}, {16, 40 043.}, {17, 40 378.9}, {18, 40 717.4}, {19, 41 058.4},
 {20, 41 401.9}, {21, 41 748.1}, {22, 42 096.9}, {23, 42 448.3}, {24, 42 802.3},
 {25, 43 159.}, {26, 43 518.3}, {27, 43 880.4}, {28, 44 245.1}, {29, 44 612.6},
 {30, 44 982.8}, {31, 45 355.8}, {32, 45 731.6}, {33, 46 110.1}, {34, 46 491.5},
 {35, 46 875.7}, {36, 47 262.8}, {37, 47 652.8}, {38, 48 045.6}, {39, 48 441.3}}
```

```
optPts = Transpose[Join[Take[Transpose[work], 1], Take[Transpose[work], {3, 3}]]]
```

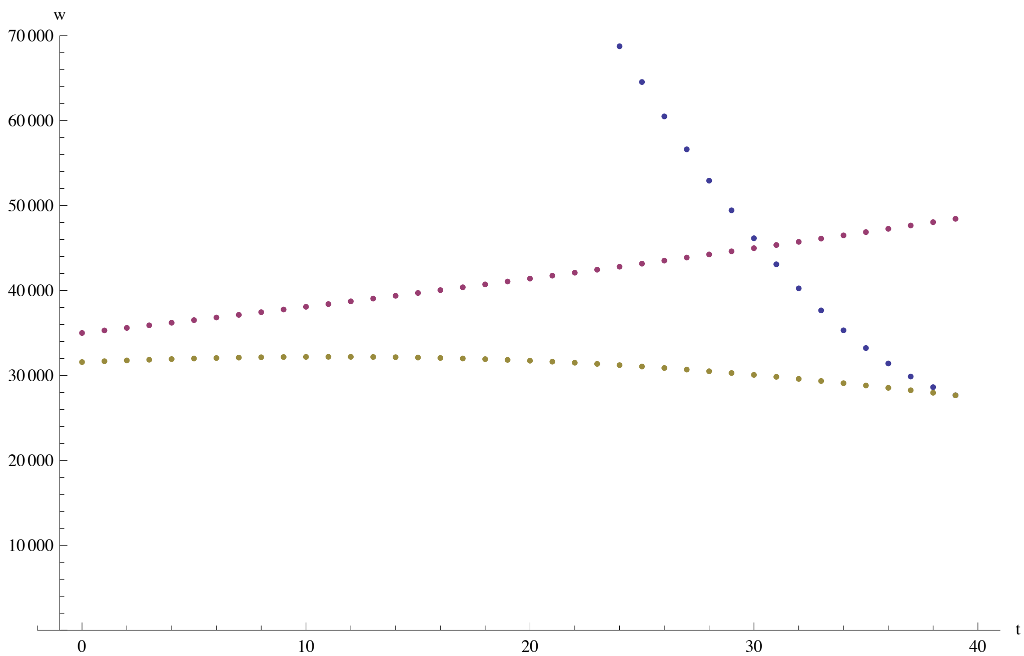
```
{{0, 31 573.2}, {1, 31 673.3}, {2, 31 765.8}, {3, 31 850.}, {4, 31 925.9},
 {5, 31 993.}, {6, 32 050.7}, {7, 32 099.2}, {8, 32 137.8}, {9, 32 166.2},
 {10, 32 184.2}, {11, 32 191.4}, {12, 32 187.6}, {13, 32 172.4}, {14, 32 145.9},
 {15, 32 107.5}, {16, 32 057.1}, {17, 31 994.7}, {18, 31 919.9}, {19, 31 832.9},
 {20, 31 733.3}, {21, 31 621.3}, {22, 31 497.}, {23, 31 360.2}, {24, 31 211.},
 {25, 31 049.5}, {26, 30 876.1}, {27, 30 690.8}, {28, 30 493.8}, {29, 30 285.5},
 {30, 30 066.1}, {31, 29 836.}, {32, 29 595.6}, {33, 29 345.2}, {34, 29 085.2},
 {35, 28 816.1}, {36, 28 538.4}, {37, 28 252.5}, {38, 27 959.1}, {39, 27 658.6}}
```

```
reservoirlevel =
 Transpose[Join[Take[Transpose[work], 1], Take[Transpose[work], {6, 6}]]]
```

```
{{0, 187 000}, {1, 182 427.}, {2, 177 753.}, {3, 172 988.}, {4, 168 138.},
 {5, 163 212.}, {6, 158 219.}, {7, 153 168.}, {8, 148 069.}, {9, 142 931.},
 {10, 137 765.}, {11, 132 581.}, {12, 127 389.}, {13, 122 202.}, {14, 117 029.},
 {15, 111 883.}, {16, 106 776.}, {17, 101 719.}, {18, 96 724.1}, {19, 91 804.2},
 {20, 86 971.3}, {21, 82 237.9}, {22, 77 616.7}, {23, 73 119.7}, {24, 68 759.5},
 {25, 64 548.5}, {26, 60 499.}, {27, 56 622.9}, {28, 52 932.1}, {29, 49 438.3},
 {30, 46 152.8}, {31, 43 086.7}, {32, 40 250.7}, {33, 37 655.1}, {34, 35 310.},
 {35, 33 224.8}, {36, 31 408.7}, {37, 29 870.3}, {38, 28 617.8}, {39, 27 658.6}}
```

generate plot

```
pl34i = ListPlot[{reservoirlevel, satiationPts, optPts},
  AxesLabel → {"t", "w"}, PlotRange → {{-2, 41}, {0, 70 000}},
  AxesOrigin → {-1, 0}]
```



Export["fig34.eps", pl34, "EPS"]

```
fig34.eps
```