You have been assigned to do a 100-year analysis of a community's strictly depletable water supply. Because you lack computer resources for a detailed 100-period treatment, your first step is to analyze two periods, each 50 years long, in a manner that closely parallels Figure 3.3 of the text.

There are two main issues in your boss's mind: how should we divide available water across time periods and how are we presently doing relative to such a goal.

Assume that household demand exhibits a constant elasticity of -0.5 so annual household demand is $w=ap^{-0.5}$ where "a" is a constant. Annual total demand is then $w=ah_tp^{-0.5}$ where h_t is the number of households.

You know h_1, the number of current households, and you are willing to assume a constant exponential population growth rate. Thus, $h_t=(1+g)^{t-1}h_1$ where g is some number like .01 (1% per year). [Hmmm, can we believe that population growth is unrelated to water availability and cost?] Your two periods might roughly center on t=30 and t=80 in terms of average population, so population in those two periods are key. The two 50-year demands are then $w_1=50ah_{30}p^{-0.5}$ and $w_2=50ah_{80}p^{-0.5}$.

Analysis to be completed:

a. Solve the latter two functions for p (invert them) and call the results MNB_1 and MNB_2 (instead of p) because you are going to assume MC=0 just to make the forthcoming math easy. Don't bother to replace h_{30} and h_{80} in these formulae until part c below.

b. To follow the Figure 3.3 analog, replace w_2 in MNB_2 with $(W-w_1)$. Set $MNB_1=MNB_2/(1+d)^{50}$, take the square root of both sides, and do the algebra needed to solve for w_1. This will tell you the proportion of total W to be used in the first 50 years. What is this proportion? How much water is available for a third period?

c. Select a discount rate and use your population growth information with a chosen g value to refine the last result. What exact shares of W do period 1 and period 2 get? Also, what is the *relative* size of the populations of the two periods (h_{30} versus h_{80})?

d. The prior assumption of MC=0 means that this entire analysis is only "suggestive" and needs to be redone. It also means that you cannot well address the boss's second issue. If p=MC=0, how much of W will people in the early period actually use? Yet, if MC>0, it is apparent that the MNB result from part (a) above is higher than the MNB actually realized in the first period, so … Please offer a paragraph of discussion about this for the boss's benefit.