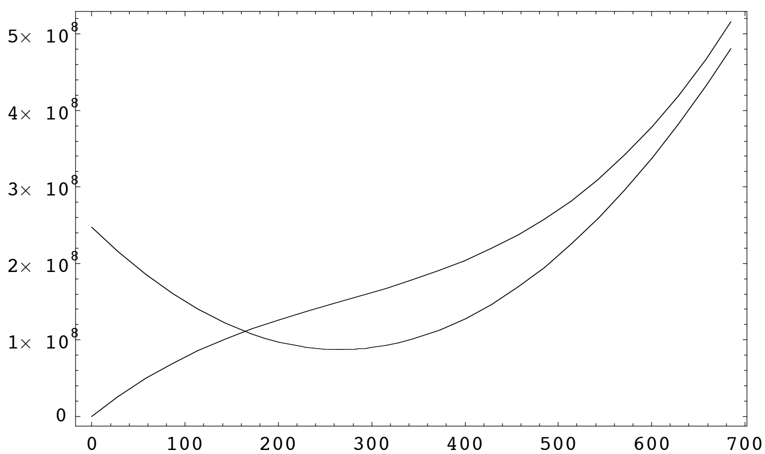
# ■ Chapter 2

```
Off[General::spell];
Off[General::spell1];
```

## ■ 2.1 Cost Functions

```
a = 3300.;
b = -8.;
c = 0.01;
tc = 300 * (a * w + b * w^2 + c * w^3);
mc = ∂_w tc;
(* s below is a scaling factor for mc, allowing mc to be plotted alongside tc *)
s = 250.;
pl21a = Plot[{tc, s * mc}, {w, 0, 685},
  Frame → True,
  DisplayFunction → $DisplayFunction
 ]
Null
```




```
- Graphics -
```

```
Solve[∂_w mc == 0, w]
```

```
{{w → 266.667}}
```

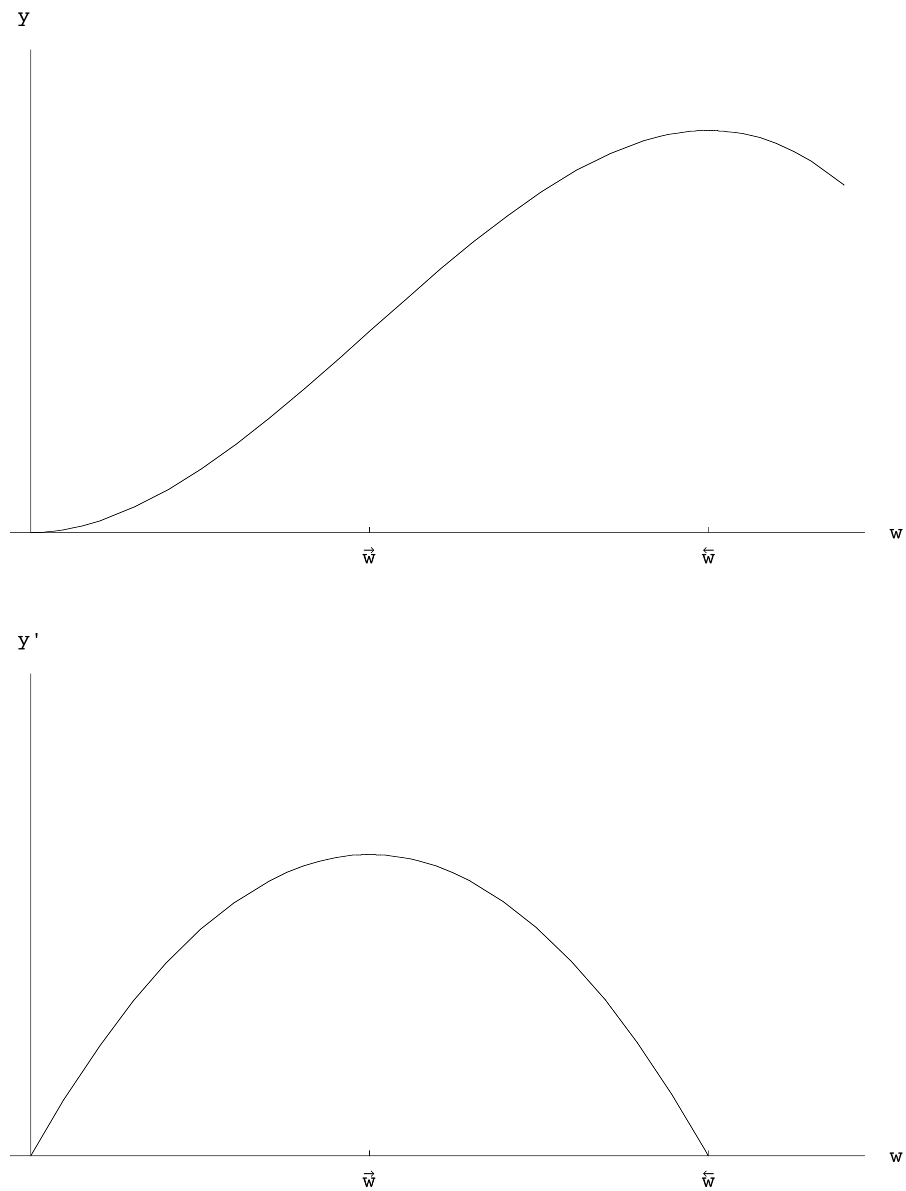
## ■ 2.2 ,2.3a,2.3b,2.4 Production Function to Demand

```
SetOptions[Plot, AspectRatio → .565];
```

```
a = 0.;
b = .003;
c = -.001;
y = a*w + b*w^2. + c*w^3;
Solve[D[y, w] == 0, w];
maxw = w /. %[[2]];
maxy = y /. w → maxw;
Solve[D[y, {w, 2}] == 0, w];
minw = w /. %[[1]];
mp = D[y, w];
maxmp = mp /. w → minw;
d1 = 0.;
d2 = .001;
mc = d1 + d2*w;
py = 1.;
vmp = py*mp;
Solve[mc - vmp == 0, w];
maxpi = w /. %[[2]];
pl22b = Plot[y, {w, 0, 1.2*maxw},
  PlotRange → {0, 1.2*maxy},
  AxesLabel → {"w", "y "},
  Ticks → {{{minw, "w⃗"}, {maxw, "w⃖"}}, {}},
  DisplayFunction → Identity
 ]
pl22d = Plot[mp, {w, 0, 1.2*maxw},
  PlotRange → {0, 1.2*maxy},
  AxesLabel → {"w", "y'"},
  Ticks → {{{minw, "w⃗"}, {maxw, "w⃖"}}, {}},
  DisplayFunction → Identity
 ]
pl22 = Show[GraphicsArray[{{pl22b}, {pl22d}}], ImageSize → 500]
```

- Graphics -

- Graphics -

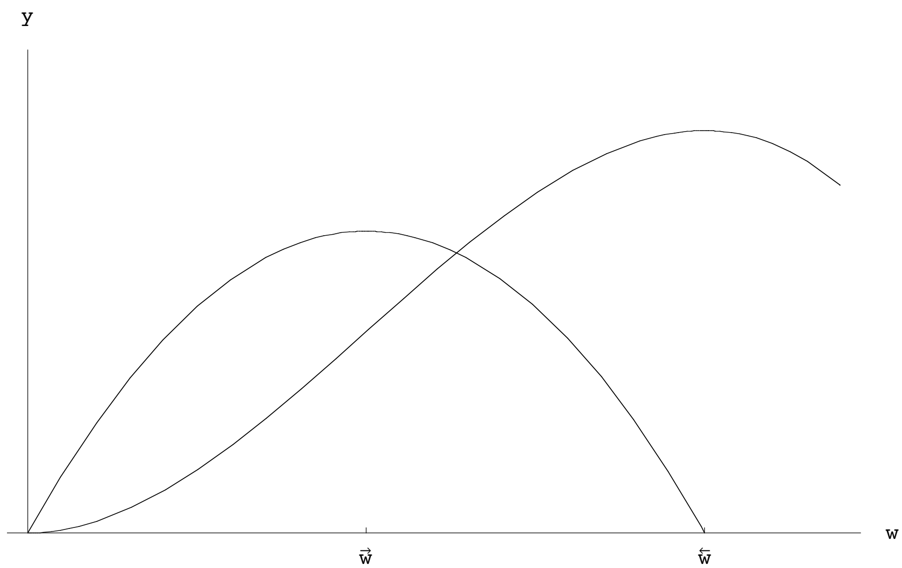


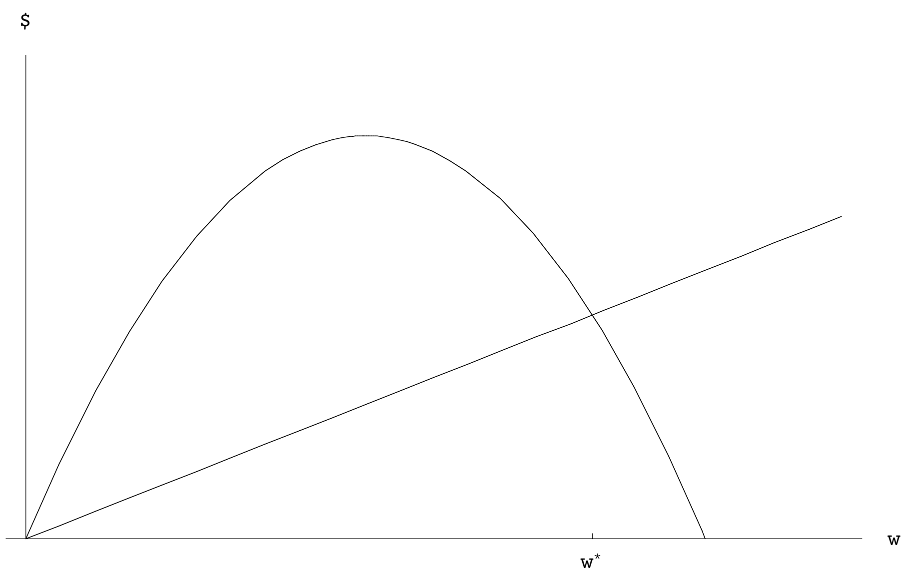

- GraphicsArray -

```
pl23b = Plot[{y, mp}, {w, 0, 1.2 * maxw},
  PlotRange → {0, 1.2 * maxy},
  AxesLabel → {"w", "y"},
  Ticks → {{{minw, "w⃗"}, {maxw, "w⃖"}}, {}}, DisplayFunction → Identity
 ]
pl23d = Plot[{mc, vmp}, {w, 0, 1.2 * maxw},
  PlotRange → {0, 1.2 * py * maxmp},
  AxesLabel → {"w", "$"},
  Ticks → {{{maxpi, "w*"}}, {}},
  DisplayFunction → Identity
 ]
pl23 = Show[GraphicsArray[{{pl23b}, {pl23d}}], ImageSize → 500]
```

- Graphics -

- Graphics -




- GraphicsArray -

```
SetOptions[Plot, AspectRatio → 1. / GoldenRatio];
```

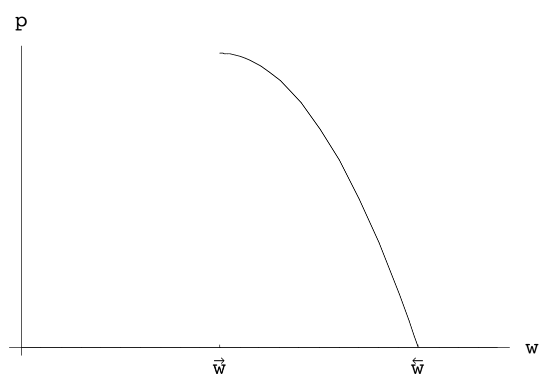
```
Solve[vmp == p, w];
wdemand = w /. %[[2]];
pdemand = p /. Flatten[Solve[w == wdemand, p]];
pl24a = Plot[{0, Which[minw ≤ w ≤ maxw, pdemand]}, {w, 0, 1.2 * maxw},
  AxesOrigin → {0, 0},
  AxesLabel → {"w", "p"},
  Ticks → {{{minw, "w⃗"}, {maxw, "w⃖"}}, {}},
  DisplayFunction → $DisplayFunction
 ]
```

```
Plot::plnr : Which[minw ≤ w ≤ maxw, pdemand] is not a machine-size real number at w = 9.999999999999998`*^-8 .

Plot::plnr : Which[minw ≤ w ≤ maxw, pdemand] is not a machine-size real number at w = 0.0973607797749979` .

Plot::plnr : Which[minw ≤ w ≤ maxw, pdemand] is not a machine-size real number at w = 0.20354111966249683` .

General::stop : Further output of Plot::plnr will be suppressed during this calculation.
```

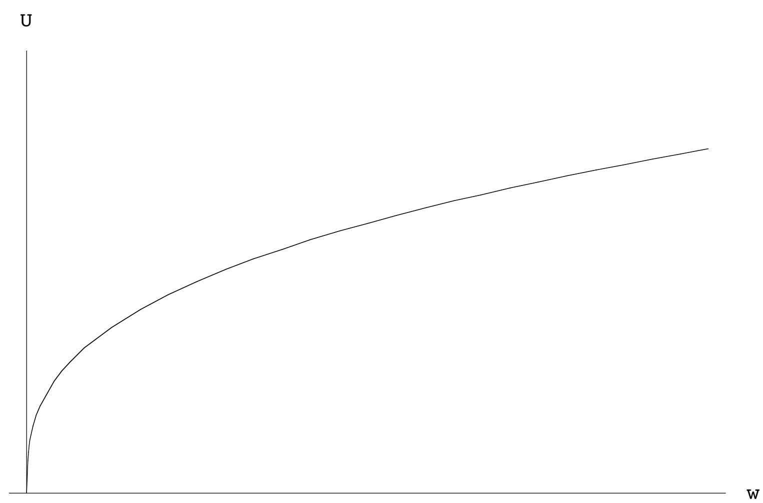



```
- Graphics -
```

## ■ 2.5 ,2.6 Utility Function to Demand

With the Cobb-Douglas utility form given below as a function of two goods, the water demand function given below will result. Max U s.t. x+pw=income with x and w as decision variables and a,b,&p as parameters.

```
w =.;
a = 0.003;
b = 0.35;
x = 2.;
income = 100.;
u = a * x * w^b;
w_d = (b * income)/(p * (1 + b));
pl25a = Plot[u, {w, 0, 20.},
  PlotRange → {0., 0.022},
  AxesLabel → {"w", "U"},
  Ticks → None,
  DisplayFunction → $DisplayFunction
 ]
```




- Graphics -

```
Solve[w_d == w, p]
```

$\{\{p \to \frac{25.9259}{w}\}\}$

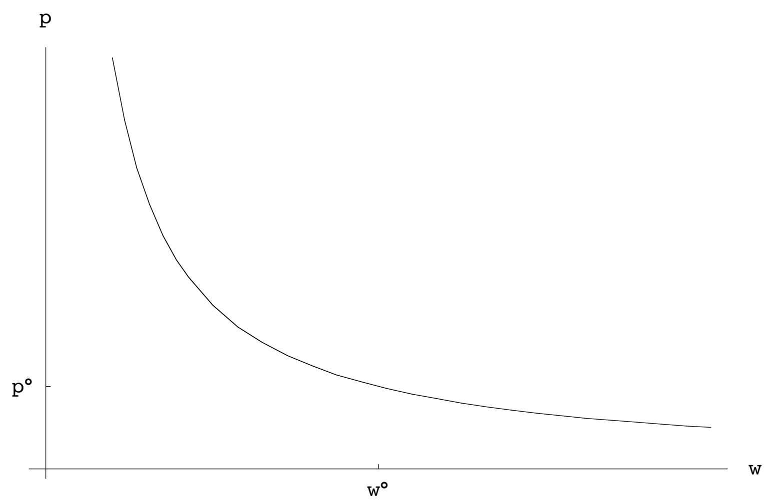
```
Solve[w_d == 10., p]
```

{{p → 2.59259}}

```
Solve[w_d == 20., p]
```

{{p → 1.2963}}

```
pl26a = Plot[25.9259 / w, {w, 2, 20},
  AxesOrigin → {0, 0},
  AxesLabel → {"w", "p"},
  Ticks → {{{10, "w°"}}, {{2.59259259, "p°"}}},
  DisplayFunction → $DisplayFunction
 ]
```




- Graphics -

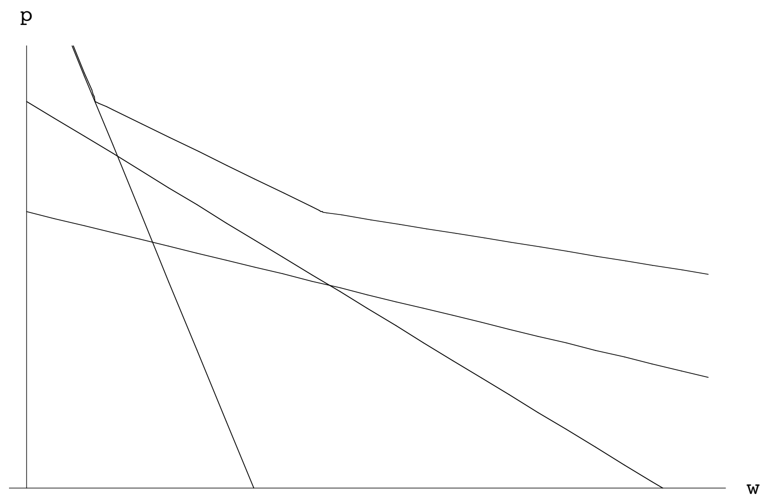
## ■ 2.7 Adding Linear Demands

```
{a,b,c,d,e,f}={10,2,7,0.5,5,0.2};
p1=a-b*w;
p2=c-d*w;
p3=e-f*w;
p=Which[0<=w<=a/b-c/b,.06+p1,
        a/b-c/b<w<a/b+c/d-e*(1/b+1/d),
        (a/b+c/d-w)/(1/b+1/d),
        a/b+c/d-e*(1/b+1/d)<=w,
        (a/b+c/d+e/f-w)/(1/b+1/d+1/f)];
pl27i=Plot[{p1,p2,p3},{w,0,15},
AxesLabel->{"w","p"},
   PlotRange->{0,8},
   Ticks->None, DisplayFunction -> Identity]
pl27ii=Plot[p,{w,0,15},
AxesLabel->{"w","p"},
   PlotRange->{0,8},
   Ticks->None, DisplayFunction -> Identity]
```

- Graphics -

- Graphics -

```
pl27 = Show[pl27i, pl27ii, DisplayFunction → $DisplayFunction]
```




```
- Graphics -
```

## ■ 2.8 Detour; Benefit Calculations

## ■ Linear

```
p =.;
w =.;
q0 = 35000.;
p0 = 3.;
elast = -0.5;
m = elast * q0 / p0;
b = q0 - m * p0;
wlin = m * p + b
plin = p /. Flatten[Simplify[Solve[w == %, p]]]
```

$52500. - 5833.33\, p$

$9. - 0.000171429\, w$

## ■ LogLinear

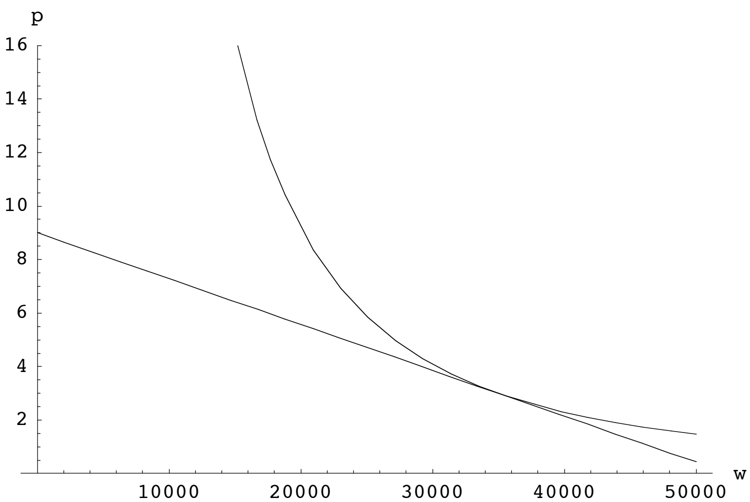
```
c = q0 / p0 ^ elast;
wlog = c * p ^ elast
plog = p /. Flatten[Simplify[Solve[w == %, p]]]
```

$\frac{60621.8}{p^{0.5}}$

$\frac{3.675 \times 10^{9}}{w^{2}}$

## ▪ Plot and NB

```
pl28i = Plot[{plin, plog}, {w, 0, q0 * 50 / 35},
  AxesLabel → {w, p},
  PlotRange → {0, 16}, DisplayFunction → $DisplayFunction]
```




- Graphics -

## ▪ Total Benefits

$$\int_0^{q0} \mathbf{plin}\, dw$$

210000.

$$\int_0^{q0} \mathbf{plog}\, dw$$

Integrate::idiv : Integral of $\frac{1}{w^2}$ does not converge on {0, 35000.}.

Integrate::idiv : Integral of $\frac{1}{w^2}$ does not converge on {0, 35000.}.

$$3.675 \times 10^9 \int_0^{35000.} \frac{1}{w^2}\, dw$$

## ▪ Gross Loss

$$\int_{0.8*q0}^{q0} \mathbf{plin}\, dw$$

25200.

$$\int_{0.8*q0}^{q0} \mathbf{plog}\, dw$$

26250.

## Net Loss

$$\int_{0.75*q0}^{q0} (\text{plin} - 3)\, dw$$

```
6562.5
```

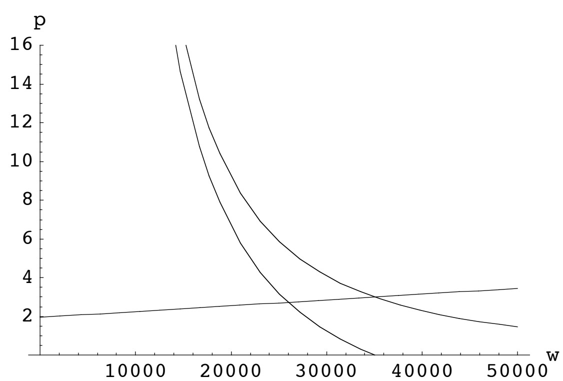
$$\int_{0.75*q0}^{q0} (\text{plog} - 3)\, dw$$

```
8750.
```

## 2.9 MNB

```
mc = 1.95 + 0.00003 * w;
mnblog = plog - mc
pl29i = Plot[{mc, plog, mnblog}, {w, 0, q0 * 50 / 35},
  AxesLabel → {w, p},
  PlotRange → {0, 16}, DisplayFunction → $DisplayFunction]
```

$$-1.95 + \frac{3.675 \times 10^{9}}{w^2} - 0.00003\, w$$




```
- Graphics -
```

## 2.10, 2.11 Two MNBs

```
plog2 = plog * 0.5;
mc2 = 1.5 + 0.00002 * w;
mnblog2 = plog2 - mc2;
flipmnblog2 = mnblog2 /. w → (50000 - w);
FindRoot[mnblog - flipmnblog2 == 0, {w, 25000}]
mnblog /. %
```

```
{w → 28056.2}
```

```
1.87707
```

■ **optimal w for 2nd sector?**

```
50000 - 28056.16
```

```
21943.8
```

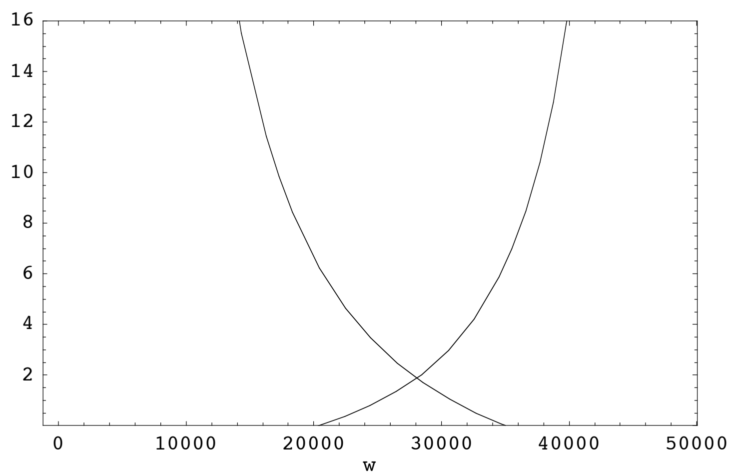
■ **MC at optimal w's for both sectors?**

```
mc /. w → 28056.2
mc2 /. w → 21943.84
```

```
2.79169
```

```
1.93888
```

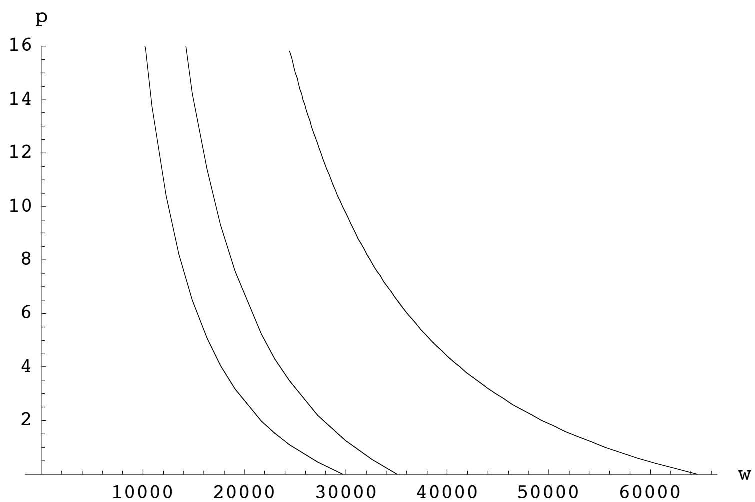
```
pl211a = Plot[{mnblog, flipmnblog2}, {w, 0, q0 * 48.8 / 35},
  FrameLabel → {"w", "", "", ""},
  PlotRange → {0, 16},
  Frame → True,
  DisplayFunction → $DisplayFunction];
```




After some work trying to add the 2 nonlinear mnb curves analytically, I gave up. They must both be inverted, and this didn't turn out well. Because elasticity is -0.5, the function to be solved for w=... is a cubic polynomial. Two roots are complex and one is real, but the real one doesn't work. It even has positive slope for both functions to be added, and I don't know why. I suspect things only work well when the degree of the polynomial is only 2, but this is pretty restrictive (elasticity must =-1, and mc must also be quadratic). So, I do it numerically below. I find w's on each curve corresponding to the same p, and then add them. I create a table of such points spanning the interesting p range. Then I connect to dots in a plot. An advantage of this procedure is that it is probably far less restrictive on MB and MC. My difficulty with doing this analytically points out an advantage of the 3-axis method at least.

```
points1 = Table[{w /. FindRoot[mnblog == p, {w, 20000}], p / 2}, {p, 0, 15.8, .2}];
points2 = Table[{w /. FindRoot[mnblog2 == p, {w, 20000}], p / 2}, {p, 0, 15.8, .2}];
points = points1 + points2;
pl210i = ListPlot[points, AxesLabel → {"w", "p"},
  PlotJoined → True, PlotRange → {0, 16}, DisplayFunction → Identity]
pl210ii = Plot[{mnblog, mnblog2}, {w, 0, 65000}, AxesLabel → {"w", "p"},
   PlotRange → {0, 16}, DisplayFunction → Identity];
pl210 = Show[{pl210i, pl210ii},
  DisplayFunction → $DisplayFunction]
```

- Graphics -




- Graphics -

## ■ 2.12 Two Sectors with One Yielding Return Flow

## ■ Method 1 - graphed

```
R = 0.25;
flipmnblog2R = mnblog2 /. w → (50000 - (1 - R) * w)
```

$$-1.5 + \frac{1.8375 \times 10^9}{(50000 - 0.75\,w)^2} - 0.00002\,(50000 - 0.75\,w)$$

```
FindRoot[mnblog / (1 - R) - flipmnblog2R == 0, {w, 25000}]
mnblog /. %
```

{w → 32407.3}

0.576995

- **Method 2 - closer to opt. model**

```
mnb1 = mnblog /. w → w1
mnb2 = mnblog2 /. w → w2
```

$$-1.95 + \frac{3.675\times 10^{9}}{w_1^2} - 0.00003\, w_1$$

$$-1.5 + \frac{1.8375\times 10^{9}}{w_2^2} - 0.00002\, w_2$$

```
FindRoot[{mnb1 == (1 - R) * mnb2, (1. - R) * w1 + w2 == 50000}, {w1, 25000}, {w2, 25000}]
```

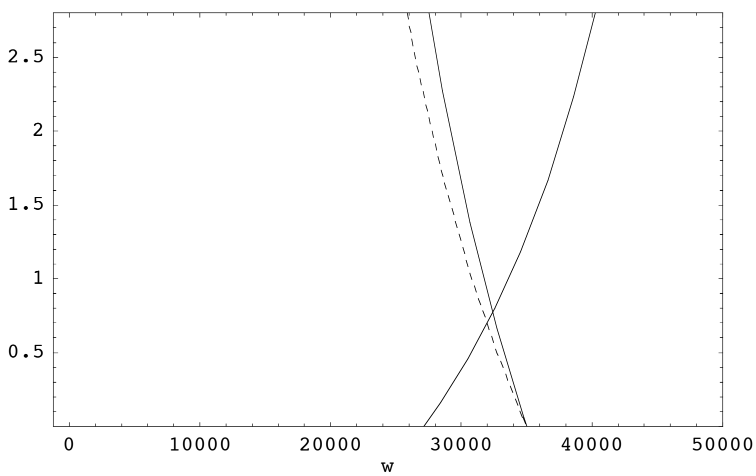
$\{w_1 \to 32407.3,\ w_2 \to 25694.5\}$

```
{mnb1, mnb2} /. %
```

{0.576995, 0.769327}

- **plot**

```
pl212a = Plot[{mnblog, mnblog / (1 - R), flipmnblog2R}, {w, 0, q0 * 48.8 / 35},
  FrameLabel → {"w", "", "", ""},
  PlotRange → {0, 2.8},
  Frame → True,
  PlotStyle → {{Dashing[{0.01, 0.01}]}, {Dashing[{1, 0}]}, {Dashing[{1, 0}]}},
  DisplayFunction → $DisplayFunction];
```




- **2.13 MNB Addition for Nonrival Uses (vs Rival)**

Assuming the a,b line has a lower p intercept (a<c) and a higher q intercept (a/b>c/d):

```
Clear[a,b,c,d]
d1=a-b*q;
d2=c-d*q;
q1=a/b-p/b;
q2=c/d-p/d;
Solve[qp==q1+q2,p]
```

$$\left\{\left\{p \to \frac{\frac{a}{b} + \frac{c}{d} - qp}{\frac{1}{b} + \frac{1}{d}}\right\}\right\}$$

```
{a,b,c,d}={6.,1,10.,3.};
nonrivalgd=Which[0<=q<=c/d,a+c-(b+d)q,
        c/d<q<=a/b,d1,
        a/b<q,0];
rivalgd=Which[0<=q<=(c-a)/d,d2,
        (c-a)/d<q<=c/d+a/b,
        (b*c+a*d-b*d*q)/(b+d)];
```

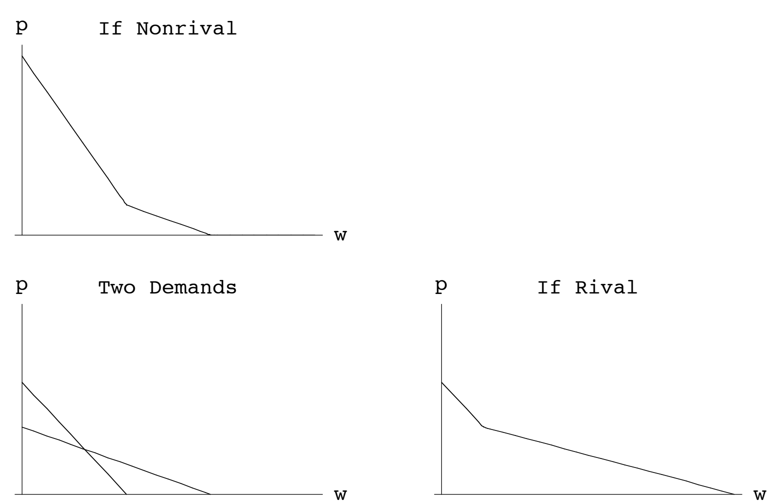
```
pl213ii = Plot[{d1, d2}, {q, 0, c / d + a / b},
  PlotRange → {0, a + c + 1}, AxesOrigin → {0, 0}, AxesLabel → {"w", "p"},
  Ticks → None, PlotLabel → "Two Demands", DisplayFunction → Identity]
pl213iv = Plot[nonrivalgd, {q, 0, c / d + a / b}, PlotRange → {0, a + c + 1},
  AxesOrigin → {0, 0}, AxesLabel → {"w", "p"}, Ticks → None,
  PlotLabel → "If Nonrival", DisplayFunction → Identity]
pl213vi = Plot[rivalgd, {q, 0, c / d + a / b}, PlotRange → {0, a + c + 1},
  AxesOrigin → {0, 0}, AxesLabel → {"w", "p"}, Ticks → None, PlotLabel → "If Rival",
  DisplayFunction → Identity]
pl213 = Show[GraphicsArray[{{pl213iv, Null},
        {pl213ii, pl213vi}}], ImageSize → 400]
```

```
- Graphics -

- Graphics -

- Graphics -

Show::gtype : Symbol is not a type of graphics.
```



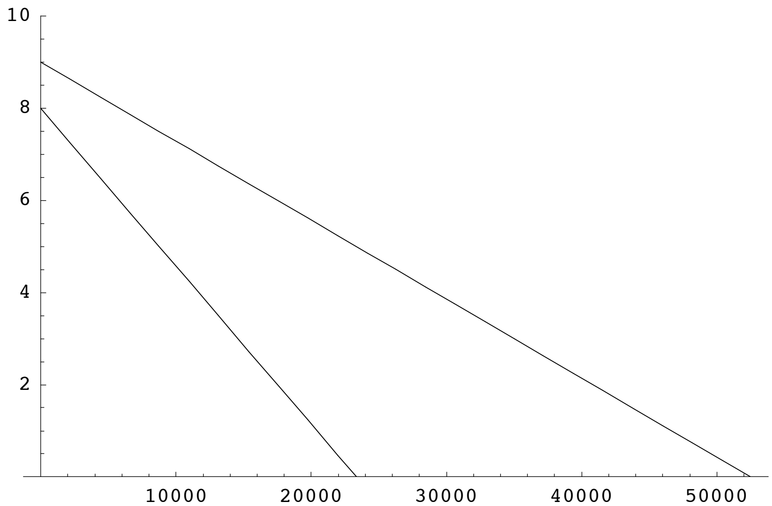

```
- GraphicsArray -
```

## ■ 2.14-2.16 Neutral Efficiency

```
plin
plin2 = Expand[plin * 2. - 10.]
Plot[{plin, plin2}, {w, 0, 52500},
 PlotRange → {0, 10}]
```

```
9. - 0.000171429 w
```

```
8. - 0.000342857 w
```




```
- Graphics -
```

Assuming the a,b line has a lower p intercept (a<c) and a higher q intercept (a/b>c/d):

```
Clear[a,b,c,d]
d1=a-b*q;
d2=c-d*q;
q1=a/b-p/b;
q2=c/d-p/d;
Solve[q*p==q1+q2,p]
```

$$\left\{\left\{p \to \frac{\frac{a}{b}+\frac{c}{d}}{\frac{1}{b}+\frac{1}{d}+q}\right\}\right\}$$

```
{a,b,c,d}={8.,0.000342857,9.,0.000171429};
dtotal=Which[0<=q<=(c-a)/d,d2,
        (c-a)/d<q<=c/d+a/b,
        (b*c+a*d-b*d*q)/(b+d)]
```

$$\text{Which}\left[0 \le q \le 5833.32,\ d2,\ \frac{c-a}{d} < q \le \frac{c}{d}+\frac{a}{b},\ \frac{b\,c+a\,d-b\,d\,q}{b+d}\right]$$

```
v = p /. Flatten[Solve[((b * c + a * d - b * d * 55000) / (b + d)) == p, p]]
```

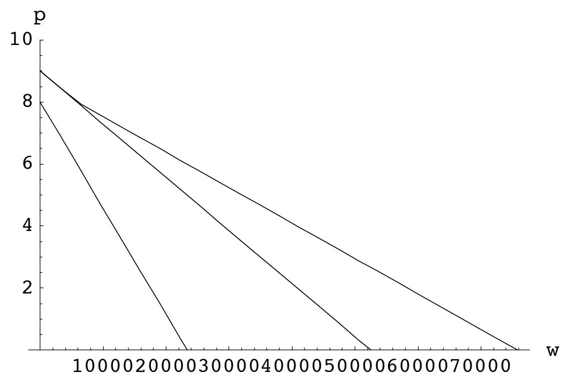
```
2.38094
```

```
wa = q1 /. p → v
wb = q2 /. p → v
```

16388.9

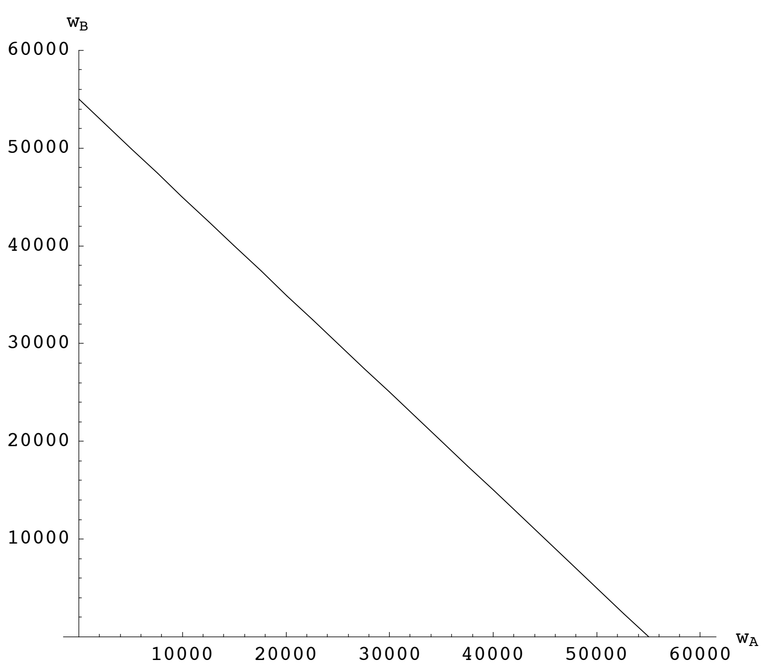
38611.1

```
pl214i = Plot[{d1, d2, dtotal}, {q, 0, c / d + a / b},
  PlotRange → {0, c + 1}, AxesOrigin → {0, 0}, AxesLabel → {"w", "p"},
  DisplayFunction → $DisplayFunction]
```




- Graphics -

```
pl215a = Plot[55000 - wa, {wa, 0, 60000},
   PlotRange → {0, 60000},
   AxesOrigin → {0, 0},
   AspectRatio → 0.9,
   AxesLabel → {"wA", "wB"},
   DisplayFunction → $DisplayFunction];
```



```
NBA = ∫_0^w d1 dq
NBB = ∫_0^w d2 dq
```

$8. w - 0.000171429 w^2$

$9. w - 0.0000857145 w^2$

```
sata = q /. Flatten[Solve[d1 == 0, q]]
satb = q /. Flatten[Solve[d2 == 0, q]]
```

23333.3

52499.9

So the Pareto frontier should only be plotted so the neither sector gets more than their satiation amounts (a's is 23333, b's is 52500).

```
NBA /. w → wa
NBA /. w → sata
NBB /. w → wb
NBB /. w → satb
```

85066.2

93333.4

219715.

236249.

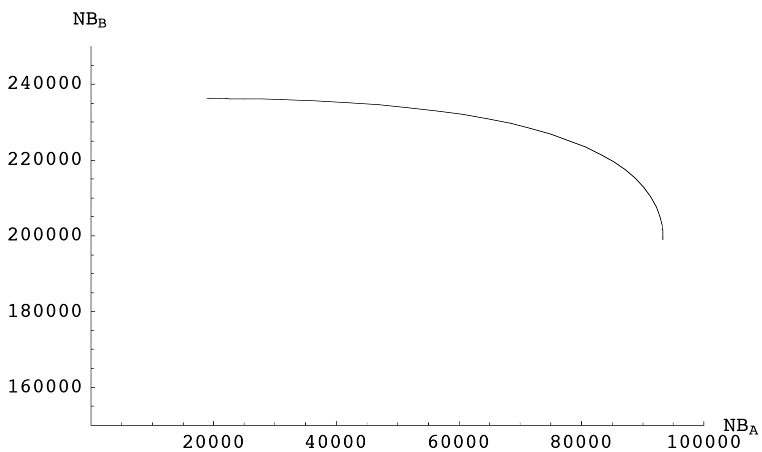
```
effpt = {85066.2, 219715.}
```

{85066.2, 219715.}

```
flipNBB = NBB /. w → (55000 - w)
```

$9. (55000 - w) - 0.0000857145 (55000 - w)^2$

```
pl216i = ParametricPlot[{NBA, flipNBB}, {w, 55000 - satb, sata},
  AxesLabel → {"NB_A", "NB_B"},
  AxesOrigin → {0, 150000},
  DisplayFunction → $DisplayFunction,
  PlotRange → {{0, 100000}, {150000, 250000}}]
```




- Graphics -